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satisfy the equation

$$\sum_{i=0}^{i=10} x^i = 0 \quad \text{or} \quad \sum_{i=1}^{i=5} \left(x^i + \frac{1}{x^i} \right) + 1 = 0. \quad (1)$$

By the substitution of y for $x + 1/x$ this equation in its second form may be reduced to¹

$$y^5 + y^4 - 4y^3 - 3y^2 + 3y + 1 = 0. \quad (2)$$

By taking $k = 1$ it will be seen that one root of (2) is

$$y = 2 \cos \frac{2\pi}{11} = 2 - 4 \sin^2 \frac{\pi}{11} = 2 - S^2, \quad \text{or} \quad -S^2 = z = y - 2.$$

Reducing by 2 the roots of (2) by synthetic division we obtain the equation for z

$$z^5 + 11z^4 + 44z^3 + 77z^2 + 55z + 11 = 0.$$

If we now substitute $-S^2$ for z and group the terms of the resulting equation we find

$$S^{10} - 22S^6 + 121S^2 = 11(S^8 - 6S^6 + 7S^4 + 6S^2 + 1)$$

which may be written

$$S^5 - 11S = \pm (S^4 - 3S^2 - 1) \sqrt{11}.$$

Since the quantity in the parenthesis is negative for $S = 2 \sin \pi/11$, the positive sign must be chosen. This gives the result

$$S^5 - (S^4 - 3S^2 - 1) \sqrt{11} - 11S = 0.$$

Also solved by T. M. BLAKSLLEE, J. S. BROWN, J. B. FAUGHT, and A. V. RICHARDSON.

2866 [1920, 482]. Proposed by NORMAN ANNING, University of Michigan.

Equilateral triangles whose sides are 1, 3, 5, 7, ... are placed so that their bases lie corner to corner in a straight line. Show that the vertices lie upon a parabola and are all at integral distances from its focus.

SOLUTION BY G. W. SMITH, University of Kansas.

If the base line of the triangles is taken as the x -axis and the y -axis is made to pass through the vertex of the first triangle of the series, the coördinates of the vertices of the triangles are $(0, \pm \frac{1}{2}\sqrt{3})$, $(2, \pm \frac{3}{2}\sqrt{3})$, $(6, \pm \frac{5}{2}\sqrt{3})$, ... The vertex of the n th triangle is given by the equations $x = n(n-1)$, $y = \pm [(2n-1)/2]\sqrt{3}$. Eliminating n from these, gives $y^2 = 3(x + \frac{1}{4})$, a parabola whose focus is $(\frac{1}{4}, 0)$. The distance from the vertex $\{n^2 - n, [(2n-1)/2]\sqrt{3}\}$ to $(\frac{1}{4}, 0)$ is $n^2 - n + 1$ which is integral. It is worth noticing above that the second difference in the abscissas of the vertices is constant while it is the first difference in the ordinates that is constant, a property of the rational integral function of the second degree, such as x is of y .

Also solved by W. E. ANDERSON, T. L. BENNETT, AUGUSTUS BOGARD, H. C. BRADLEY, E. F. CANADAY, P. J. DA CUNHA, J. F. DOVE, L. H. DUBE, W. C. EELLS, J. B. FAUGHT, MICHAEL GOLDBERG, A. M. HARDING, LESTA HOEL, G. R. LIVINGSTON, GERTRUDE I. MCCAIN, I. MAIZLISH, E. W. MARTIN, L. C. MATHEWSON, H. L. OLSON, ARTHUR PELLETIER, R. V. PRITCHARD, R. V. RICHARDSON, and T. O. WALTON.

¹ See Burnside and Panton, *The Theory of Equations*, eighth edition, vol. 1, 1918, p. 90 where the necessary reductions are given; on page 100, ex. 3 the resulting equation (2) is also given.